

Holography of 3D asymptotically flat black holes

Reza Fareghbal^{a,b,*} and Seyed Morteza Hosseini^{a,c,d,†}

^a*Department of Physics, Shahid Beheshti University G.C., Evin, Tehran 19839, Iran*

^b*School of Particles and Accelerators,*

Institute for Research in Fundamental Sciences (IPM),

P.O. Box 19395-5531, Tehran, Iran

^c*Dipartimento di Fisica, Università di Milano-Bicocca, I-20126 Milano, Italy and*

^d*INFN, sezione di Milano-Bicocca, I-20126 Milano, Italy*

Abstract

We study the asymptotically flat rotating hairy black hole solution of a three-dimensional gravity theory which is given by taking the flat-space limit (zero cosmological constant limit) of new massive gravity. We propose that the dual field theory of the flat-space limit of new massive gravity can be described by a contracted conformal field theory which is invariant under the action of the BMS_3 group. Using the flat/contracted conformal field theory correspondence, we construct a stress tensor which yields the conserved charges of the asymptotically flat black hole solution. We check that our expressions of the mass and angular momentum fit with the first law of black hole thermodynamics. Furthermore, by taking the appropriate limit of the Cardy formula in the parent conformal field theory, we find a Cardy-like formula which reproduces the Wald's entropy of the 3D asymptotically flat black hole.

PACS numbers: 04.60.Kz, 11.25.Tq

*Electronic address: r_fareghbal@sbu.ac.ir

†Electronic address: morteza.hosseini@mib.infn.it

Contents

1. Introduction	2
2. Bulk Solutions	5
2.1. Rotating hairy black hole of NMG	5
2.2. The flat-space limit of NMG and its black hole solution	6
3. Dual Boundary Theory	8
3.1. CFT dual to NMG	8
3.2. CCFT dual to the flat-space limit of NMG	9
3.2.1. Symmetries of CCFT	9
3.2.2. Quasi local stress tensor	11
3.2.3. Cardy-like formula	12
4. Conclusions	13
Acknowledgments	14
References	15

1. INTRODUCTION

Taking the flat-space limit (zero cosmological constant limit) of asymptotically AdS spacetimes results in asymptotically flat geometries. This procedure can be done by taking the $\ell \rightarrow \infty$ limit where ℓ is the radius of AdS spacetime. From the field theory perspective, one could expect that the $\ell \rightarrow \infty$ limit in the bulk theory has a holographic description at the boundary. Recently, it has been argued that the flat-space limit of AdS gravity is dual to the İnönü-Wigner contraction of the boundary conformal field theory (CFT) [1, 2].¹

The so-called flat/contracted conformal field theory (CCFT) correspondence has received a great deal of attention recently. For example, in [4] a Cardy-like formula has been obtained

¹ It is worth noticing that another interesting approach to understanding flat space quantum gravity is given in [3]. Therein, the authors showed that interpreting the inverse AdS₃ radius $1/\ell$ as a Grassmann variable leads to a map from gravity in AdS₃ to gravity in flat space.

for the two-dimensional CCFT which yields the correct entropy of the three-dimensional cosmological solution. These asymptotically flat spacetimes can be obtained by taking the flat-space limit, as in [5], of nonextremal Bañados-Teitelboim-Zanelli (BTZ) black holes. After taking the flat-space limit, the outer horizon of BTZ is mapped to infinity; however, the value of the inner horizon remains finite and defines the cosmological horizon. The entropy of the cosmological solution has been identified with the area of the cosmological horizon. In the literature (see for example [6]), a modified Cardy formula has been introduced which reproduces the entropy of the inner horizon of BTZ black holes. The CFT origin of this formula has not been well understood yet, but the observation of [7, 8] is that if we accept the modified Cardy formula related to the inner horizon of the BTZ and contract it by using appropriate parameters of CCFT, the final result is exactly the CCFT Cardy-like formula which yields the entropy of the cosmological horizon.

Furthermore, in [9], the authors found the correlation functions of CCFT energy-momentum tensor by using the contraction of CFT correlation functions for finding the quasilocal stress tensor of the asymptotically flat spacetimes which gives the correct conserved charges of these geometries.

The Flat/CCFT correspondence can also propose a dual field theory which lives at the horizon of nonextreme black holes. The idea begins from the appearance of Rindler spacetime in the near horizon limit of nonextreme black holes. If one starts with the Rindler-AdS/CFT correspondence [10, 11] and takes the flat-space limit in the bulk, which results in the Rindler spacetime, the boundary field theory is given by the contraction of the parent CFT. This proposal has been used in [12]. Therein, the authors found the Bekenstein-Hawking entropy of the nonextreme BTZ black hole by counting the CCFT microstates.

Moreover, Bagchi *et al.* calculated the entanglement entropy of a two-dimensional field theory with Galilean conformal symmetry² recently [13]. The authors used the Wilson lines approach and found the holographic entanglement entropy they computed is in precise agreement with the ones obtained in the field theory side. For an almost complete list of papers related to the Flat/CCFT correspondence, see the references of [13].

In (2+1)-dimensional Einstein gravity, black holes can exist only in the presence of a

² The group of symmetries of CCFT₂ is isomorphic to Galilean Conformal Algebra but in higher dimensions these are not the same.

negative cosmological constant [14]. In order to find asymptotically flat black holes in three dimensions one has to consider higher-derivative gravity theories. The entropy of these black holes can be obtained by Wald's formula [15]. A successful theory of quantum gravity should be able to give a microscopic description of this semiclassical entropy. An alternative approach for this study is using holography. The Flat/CCFT correspondence as a duality between quantum gravity in the asymptotically flat backgrounds and a field theory with contracted conformal symmetry can be an appropriate context to study this problem. The current paper is focused in this direction.

We consider a theory of gravity which is given by taking the flat-space limit of new massive gravity (NMG) [16]. This theory possesses remarkable properties. In [17], it was shown that it is a ghost-free and power-counting UV finite, three-dimensional gravity. We use the dictionary of the Flat/CCFT correspondence for finding quasi local stress tensor of the new type of asymptotically flat black hole [18]. Using the holographic stress tensor along with the Brown and York's method [19], we compute the conserved charges of this black hole. We also take the limit from the Cardy formula and find a Cardy-like formula for the CCFT and show that this gives agreement with the Wald's semiclassical approach.

The next sections are devoted to two main parts. In Sec.(2) we introduce the bulk solutions. We start from NMG and review its asymptotically AdS rotating hairy black hole. Then we take the flat-space limit from the action and its black hole solution and introduce the asymptotically flat rotating hairy black hole with some novel properties. We calculate its entropy using Wald's formula and verify for it the first law of black hole thermodynamics. In Sec.(3) we argue about the dual boundary theory of the bulk solution. We shortly review the known results about the dual CFT of NMG and then try to contract these results and find a CCFT which is dual to the higher-derivative gravity theory of [17]. This work is another check for the correctness of the Flat/CCFT correspondence.

2. BULK SOLUTIONS

2.1. Rotating hairy black hole of NMG

We consider the three-dimensional higher-derivative gravity theory known as NMG. This theory is described by the parity-invariant action [16]

$$S = \frac{1}{16\pi G} \int d^3x \sqrt{-g} \left[R - 2\Lambda + \frac{1}{m^2} K \right], \quad (2.1)$$

where

$$K = R_{\mu\nu} R^{\mu\nu} - \frac{3}{8} R^2. \quad (2.2)$$

The above theory (2.1) for the special case $m^2 \ell^2 = 1/2$ has the following rotating black hole solution [18]

$$ds^2 = -NF dt^2 + \frac{dr^2}{F} + r^2 (d\phi + N^\phi dt)^2, \quad (2.3)$$

where N , N^ϕ , and F are given by

$$\begin{aligned} N &= \left[1 + \frac{b\ell^2}{4H} \left(1 - \Xi^{\frac{1}{2}} \right) \right]^2, \\ N^\phi &= -\frac{a}{2r^2} (4GM - bH), \\ F &= \frac{H^2}{r^2} \left[\frac{H^2}{\ell^2} + \frac{b}{2} \left(1 + \Xi^{\frac{1}{2}} \right) H + \frac{b^2 \ell^2}{16} \left(1 - \Xi^{\frac{1}{2}} \right)^2 - 4GM \Xi^{\frac{1}{2}} \right], \end{aligned} \quad (2.4)$$

and

$$\begin{aligned} H &= \left[r^2 - 2GM\ell^2 \left(1 - \Xi^{\frac{1}{2}} \right) - \frac{b^2 \ell^4}{16} \left(1 - \Xi^{\frac{1}{2}} \right)^2 \right]^{\frac{1}{2}}, \\ \Xi &= 1 - a^2/\ell^2. \end{aligned} \quad (2.5)$$

It is labeled by three parameters: the mass M , the angular momentum $J = Ma$, and an additional “gravitational hair” parameter b . The rotation parameter a is bounded according to $-\ell \leq a \leq \ell$.

The angular velocity of the horizon is

$$\Omega_+ = \frac{1}{a} \left(\Xi^{\frac{1}{2}} - 1 \right). \quad (2.6)$$

We can associate a Hawking temperature and entropy to it

$$T = \frac{\Xi^{\frac{1}{2}}}{\pi\ell} \sqrt{2G\Delta M \left(1 + \Xi^{\frac{1}{2}} \right)^{-1}}, \quad (2.7)$$

$$S = \pi\ell \sqrt{\frac{2}{G} \Delta M \left(1 + \Xi^{\frac{1}{2}} \right)}, \quad (2.8)$$

where

$$\Delta M = M + \frac{b^2 \ell^2}{16G} . \quad (2.9)$$

These quantities fulfill the relation

$$TdS = \Xi^{\frac{1}{2}} dM + \frac{b\ell^2}{8G} \Xi^{\frac{1}{2}} db - \frac{1}{a} \left(1 - \Xi^{\frac{1}{2}}\right) \Delta M da . \quad (2.10)$$

Not much more work would bring this equation to the familiar form of the first law of black hole thermodynamics

$$d(\Delta M) = TdS - \Omega_+ d(\Delta J) , \quad (2.11)$$

where we defined

$$\Delta J = a\Delta M . \quad (2.12)$$

2.2. The flat-space limit of NMG and its black hole solution

In order to have a well-defined flat-space limit ($\Lambda \rightarrow 0$ or $\ell \rightarrow \infty$) for (2.1) in the special point $m^2 \ell^2 = 1/2$, we also need to scale Newton's constant to infinity while keeping fixed $\kappa = \ell^2/G$. Thus, the flat-space limit of NMG action (2.1) becomes

$$S = \frac{\kappa}{8\pi} \int d^3x \sqrt{-g} K. \quad (2.13)$$

Moreover, a well-defined flat-space limit for the black hole solution (2.3) needs a scaling of the mass parameter M such that $\mu = MG$ remains fixed. The final line element for the asymptotically flat rotating hairy black hole is given by

$$ds^2 = -\mathcal{F} dt^2 + \frac{r^2}{\mathcal{F}\Delta} dr^2 + a\mathcal{F} dt d\phi + r^2 d\phi^2, \quad (2.14)$$

where $\Delta(r)$ and $\mathcal{F}(r)$ are functions of the radial coordinate r , given by

$$\begin{aligned} \Delta &= r^2 - \mu a^2 - \left(\frac{a^2 b}{8}\right)^2, \\ \mathcal{F} &= b\sqrt{\Delta} - 4\mu. \end{aligned} \quad (2.15)$$

The Ricci scalar of this asymptotically flat black hole can be written as

$$R = -\frac{16b}{a^2 b + 8\sqrt{\Delta}}. \quad (2.16)$$

One can verify that (2.14) satisfies the equations of motion resulting from the action (2.13). It is worth noting that in [17], it was argued that the three-dimensional gravity theory described by (2.13) is ghost free and finite.

Horizons of (2.14) are at

$$r_+ = \frac{a^2 b}{8} + \frac{4\mu}{b}, \quad r_- = \sqrt{\left(\frac{a^2 b}{8}\right)^2 + \mu a^2}, \quad (2.17)$$

and one can calculate the entropy of the outer horizon using the Wald's formula. This formula gives the black hole entropy in an arbitrary diffeomorphism invariant theory and is given by

$$S = -\frac{2\pi}{16\pi G} \int_{\Sigma_h} \frac{\delta L}{\delta R_{\alpha\beta\gamma\delta}} \epsilon_{\alpha\beta} \epsilon_{\gamma\delta} \bar{\epsilon}, \quad (2.18)$$

where L is the Lagrangian, and $\bar{\epsilon}$, $\epsilon_{\mu\nu}$, denote the volume form and the binormal vector to the spacelike bifurcation surface Σ_h , respectively. $\epsilon_{\mu\nu}$ is normalized as $\epsilon^{\mu\nu} \epsilon_{\mu\nu} = -2$. For the action (2.13) and the asymptotically flat rotating hairy black hole solution (2.14) we obtain

$$\frac{\partial L}{\partial R_{\alpha\beta\gamma\delta}} = \frac{3}{8} R (g^{\alpha\delta} g^{\beta\gamma} - g^{\alpha\gamma} g^{\beta\delta}) + \frac{1}{2} (g^{\alpha\gamma} R^{\beta\delta} - g^{\alpha\delta} R^{\beta\gamma} - g^{\beta\gamma} R^{\alpha\delta} + g^{\beta\delta} R^{\alpha\gamma}), \quad (2.19a)$$

$$\epsilon_{\alpha\beta} = -\left(\frac{a^2 \mathcal{F} + 4r^2}{\Delta}\right)^{\frac{1}{2}} \delta_{[\alpha}^t \delta_{\beta]}^r. \quad (2.19b)$$

Therefore, the Wald's entropy for the new type of asymptotically flat black hole becomes

$$S_{\text{flat}} = \frac{\pi \kappa b}{2}. \quad (2.20)$$

It is instructive to derive the above entropy by taking the flat-space limit of the entropy (2.8). The entropy can, therefore, be computed as follows:

$$\begin{aligned} \lim_{\ell \rightarrow \infty} S &= \lim_{\ell \rightarrow \infty} \pi \ell \sqrt{\frac{2}{G} \Delta M \left(1 + \Xi^{\frac{1}{2}}\right)} \\ &= \lim_{\ell \rightarrow \infty} \frac{\pi \kappa b}{2} \sqrt{1 + \frac{16\mu}{b^2 \ell^2}} \\ &= \frac{\pi \kappa b}{2} = S_{\text{flat}}. \end{aligned} \quad (2.21)$$

Now, consider the Hawking temperature. From (2.7) it follows that

$$\begin{aligned} \lim_{\ell \rightarrow \infty} T &= \lim_{\ell \rightarrow \infty} \frac{1}{\pi \ell} \Xi^{\frac{1}{2}} \sqrt{2G \Delta M \left(1 + \Xi^{\frac{1}{2}}\right)^{-1}}, \\ &= \lim_{\ell \rightarrow \infty} \frac{b}{4\pi} \sqrt{1 + \frac{16\mu}{b^2 \ell^2}} \\ &= \frac{b}{4\pi} = T_{\text{flat}}. \end{aligned} \quad (2.22)$$

These quantities fulfill the relation

$$T_{\text{flat}} dS_{\text{flat}} = \frac{b\kappa}{8} db. \quad (2.23)$$

This agrees precisely with the $\ell \rightarrow \infty$ limit of (2.10). A direct calculation by using (2.14) or taking the flat-space limit of (2.6) shows that the angular velocity of the black hole (2.14) at the outer horizon is zero ($\Omega_{+\text{flat}} = 0$) though it has a nonvanishing angular momentum.

From (2.20) and (2.22), it is clear that the hair parameter b determines the entropy and the temperature of the outer horizon. In the $b \rightarrow 0$ limit the hairy black hole (2.14) is reduced to the cosmological solution of [5]. In this limit, r_+ is mapped to infinity; however, r_- remains finite and defines the cosmological horizon.

3. DUAL BOUNDARY THEORY

3.1. CFT dual to NMG

In [20, 21], it was proposed that NMG has a dual description in terms of a CFT.³ The charges associated to the asymptotic symmetries enhance the isometry of asymptotically AdS_3 spacetimes to two copies of the Virasoro algebra. The central charges are given by

$$c_{\pm} = c = \frac{3\ell}{2G} \left(1 + \frac{1}{2m^2\ell^2} \right). \quad (3.1)$$

At the spacial point $m^2\ell^2 = 1/2$, the central charges are twice the values proposed by Brown and Henneaux for the Einstein gravity with negative cosmological constant [23], i.e.,

$$c = \frac{3\ell}{G}. \quad (3.2)$$

The entropy of the black hole (2.3) can be given by the Cardy formula

$$S = 2\pi\sqrt{\frac{c_+\Delta_+}{6}} + 2\pi\sqrt{\frac{c_-\Delta_-}{6}}, \quad (3.3)$$

where Δ_{\pm} are the eigenvalues of the left and right Virasoro generators L_0^{\pm} and are given by

$$\Delta_{\pm} = \frac{1}{2}\Delta M(\ell \pm a). \quad (3.4)$$

³ In 2013, de Buyl *et al.* considered the asymptotically dS case, $\Lambda = +1/\ell^2 > 0$, within the context of dS/CFT correspondence [22]. We would like to thank the referee for bringing this paper to our attention.

Using (3.2) and (3.4), this is

$$S = \pi\ell\sqrt{\frac{2}{G}\left(1 + \Xi^{\frac{1}{2}}\right)\Delta M}, \quad (3.5)$$

in precise agreement with (2.8).

3.2. CCFT dual to the flat-space limit of NMG

In this section we want to propose a dual description for the theory of gravity given by (2.13). To do so, we will use the idea which was first proposed in papers [1, 2]. That is, if we start from the AdS/CFT correspondence, the large AdS radius limit in the bulk is equivalent to a contraction of spacetime coordinates in the boundary CFT.⁴

We shall first show how one obtains the appropriate coordinate which must be contracted in the parent CFT. Let us look at the conformal boundary of the black hole (2.3) for an arbitrary large ℓ . It could be written as follows:

$$ds_{\text{C.B.}}^2 = \frac{r^2}{\kappa^2} \left(-\frac{\kappa^2}{\ell^2} dt^2 + \kappa^2 d\phi^2 \right). \quad (3.6)$$

We have used κ in the conformal factor to make it dimensionless. Moreover, the fact that κ is fixed in our flat-space limit makes the conformal factor well defined for all large values of ℓ . Now, ℓ can be absorbed by defining new time coordinate as $\tau = \kappa t/\ell$. The dual CFT lives on a cylinder with coordinates (τ, ϕ) and radius κ . Taking the $\ell \rightarrow \infty$ limit (or $\kappa/\ell \rightarrow 0$ limit), it is obvious that the flat-space limit in the bulk induces a contraction in t of the boundary CFT reducing it to the two-dimensional CCFT.

3.2.1. Symmetries of CCFT

According to the proposal of [1, 2], the symmetries of CCFT realize the group of asymptotic symmetries of the asymptotically flat spacetimes at null infinity, namely the Bondi-Metzner-Sachs (BMS) group [24, 25]. There is a very precise procedure, called the İnönü-Wigner contraction, by which one can obtain the CCFT algebra from the relativistic con-

⁴ We refer to [2] for a full-scale investigation into the CCFT representation.

formal algebra of the parent CFT. Let us consider two copies of the Virasoro algebra⁵

$$\begin{aligned}[L_m^+, L_n^+] &= (m-n)L_{n+m}^+ + \frac{c^+}{12}m(m^2-1)\delta_{m+n,0}, \\ [L_m^-, L_n^-] &= (m-n)L_{n+m}^- + \frac{c^-}{12}m(m^2-1)\delta_{m+n,0}, \\ [L_m^+, L_n^-] &= 0.\end{aligned}\tag{3.7}$$

For a small parameter ϵ , at the level of the algebra, if we define⁶

$$\begin{aligned}L_n &= L_n^+ - L_{(-n)}^-, \\ M_n &= \epsilon \left(L_n^+ + L_{(-n)}^- \right),\end{aligned}\tag{3.8}$$

we can see that the CCFT algebra is generated from the Virasoro algebras, on taking the $\epsilon \rightarrow 0$ limit, i.e.,

$$\begin{aligned}[L_m, L_n] &= (m-n)L_{n+m} + \frac{c_L}{12}m(m^2-1)\delta_{m+n,0}, \\ [L_m, M_n] &= (m-n)M_{n+m} + \frac{c_M}{12}m(m^2-1)\delta_{m+n,0}, \\ [M_m, M_n] &= 0,\end{aligned}\tag{3.9}$$

where the central charges c_L and c_M are given by the linear combination of the parent relativistic central charges

$$c_L = \lim_{\epsilon \rightarrow 0}(c^+ - c^-), \quad c_M = \lim_{\epsilon \rightarrow 0}\epsilon(c^+ + c^-).\tag{3.10}$$

The algebra (3.9) which is given by the contraction of the Virasoro algebra in the boundary theory is exactly the (centrally extended) BMS₃ algebra [26].

We would expect the same symmetry group for the CCFT dual to the theory described by the action (2.13) at the special point $m^2\ell^2 = 1/2$. As we stated earlier, the $\epsilon \rightarrow 0$ limit in the boundary corresponds to the flat-space limit or, more precisely, the $\kappa/\ell \rightarrow 0$ limit in the bulk side. Using (3.2) and (3.10), for the problem in hand we find

$$c_L = c_M = 0.\tag{3.11}$$

⁵ The relativistic conformal algebra consists of two copies of the Virasoro algebra.

⁶ It is clear from (3.8) that other positive powers of ϵ are meaningless since one can redefine ϵ and rewrite them finally as in (3.8). This definition is consistent with the observation of [26]. That is, one can obtain the BMS₃ algebra by taking the flat-space limit from the asymptotic symmetry algebra of three-dimensional asymptotically AdS spacetimes.

We will show that although the CCFT algebra has vanishing central charges, it is possible to find a Cardy-like formula for the asymptotic growth of the number of states which reproduces the entropy of the black hole (2.14). To add strength to this claim, let us find more evidence about the correctness of our proposal using the CCFT energy-momentum tensor.

3.2.2. Quasi local stress tensor

The one-point function of the CCFT energy-momentum operator corresponds to the quasilocal stress tensor of the bulk theory. It was argued in [9] that the definition (3.8) provides a recipe to calculate the components of the stress tensor in the asymptotically flat spacetimes. Therefore, we can write

$$\begin{aligned}\tilde{T}_{++} + \tilde{T}_{--} &= \lim_{\epsilon \rightarrow 0} \epsilon (T_{++} + T_{--}) , \\ \tilde{T}_{++} - \tilde{T}_{--} &= \lim_{\epsilon \rightarrow 0} (T_{++} - T_{--}) , \\ \tilde{T}_{+-} &= \lim_{\epsilon \rightarrow 0} T_{+-} ,\end{aligned}\tag{3.12}$$

where T_{ij} and \tilde{T}_{ij} are, respectively, the stress tensor of the asymptotically AdS and flat spacetimes and x^\pm are the light-cone coordinates constructed from the nonradial coordinates of the metrics. In the above definition it was assumed that both the asymptotically AdS and flat spacetimes are given in the BMS gauge [9].

The nonzero components of the stress tensor at the boundary of the asymptotically AdS black hole (2.3) are given by [27]

$$\begin{aligned}T_{tt} &= \frac{1}{8\pi G\ell} \left(\frac{b^2\ell^2}{4} + 4MG \right) , \\ T_{t\phi} &= -\frac{a}{8\pi G\ell} \left(\frac{b^2\ell^2}{4} + 4MG \right) , \\ T_{\phi\phi} &= \frac{\ell}{8\pi G} \left(\frac{b^2\ell^2}{4} + 4MG \right) .\end{aligned}\tag{3.13}$$

The formula (3.12) results in a stress tensor \tilde{T}_{ij} for the asymptotically flat black hole (2.14) as follows:

$$\tilde{T}_{tt} = \frac{b^2}{32\pi} , \quad \tilde{T}_{\phi\phi} = \frac{\kappa^2 b^2}{32\pi} , \quad \tilde{T}_{t\phi} = -\frac{ab^2}{32\pi} .\tag{3.14}$$

Using \tilde{T}_{ij} we can calculate the conserved charges of the black hole (2.14).

Let us denote the hypersurface of the spacetime where CCFT lives with $\partial\mathcal{M}$. Its line element is given by taking the $\ell \rightarrow \infty$ limit of the conformal boundary (3.6),

$$ds_{\partial\mathcal{M}}^2 = \frac{r^2}{\kappa^2} (-dt^2 + \kappa^2 d\phi^2). \quad (3.15)$$

Following Brown and York's method [19], the charges associated to a boundary Killing vector ξ^μ are given by

$$Q_\xi = \int_\Sigma d\phi \sqrt{\sigma} \xi^\mu n^\nu \tilde{T}_{\mu\nu}, \quad (3.16)$$

where Σ is the spacelike surface embedded in $\partial\mathcal{M}$ with induced metric $\sigma_{\mu\nu}$. Moreover, n^μ is the timelike unit normal to Σ . Using (3.15) and (3.16), the mass and the angular momentum of the asymptotically flat black hole (2.14) are

$$\mathcal{M} = Q_{\partial_t} = \frac{\kappa b^2}{16}, \quad \mathcal{J} = Q_{\partial_\phi} = -\frac{\kappa a b^2}{16}. \quad (3.17)$$

It is clear that $|\mathcal{J}|/\mathcal{M} = a$ as expected.

Given the expressions above, together with (2.23), it is straightforward to check that the first law of black hole thermodynamics is satisfied, i.e.

$$d\mathcal{M} = T_{\text{flat}} dS_{\text{flat}} - \Omega_{+\text{flat}} d\mathcal{J}. \quad (3.18)$$

3.2.3. Cardy-like formula

If the gravity theory (2.13) has a dual description in terms of a CCFT, then the entropy of the black hole (2.14) must be given by the asymptotic growth of the number of states in the boundary theory. In [4], the authors found a Cardy-like formula by computing the CCFT partition function using the saddle-point approximation. However, in the recent papers [7, 8] it was shown that the Cardy-like formula of [4] can be obtained if one writes the Cardy formula in terms of CCFT parameters and then takes the $\epsilon \rightarrow 0$ limit. In the current work we will use the same approach and take the $\epsilon \rightarrow 0$ limit from the Cardy formula in the parent CFT.

The CCFT algebra is given by (3.9). We denote the eigenvalues of L_0 and M_0 by Δ_L and Δ_M , respectively. For the current problem $c_L = c_M = 0$, however, the eigenvalues of L_0 and M_0 are nonzero. From the viewpoint of the limit (3.8) we see that the two labels Δ_L and

Δ_M are related to the conformal weights in the two-dimensional CFT as ⁷

$$\Delta_L = \lim_{\epsilon \rightarrow 0} (\Delta_+ - \Delta_-) , \quad \Delta_M = \lim_{\epsilon \rightarrow 0} \epsilon (\Delta_+ + \Delta_-) . \quad (3.19)$$

We would like to remind the reader that the $\epsilon \rightarrow 0$ limit in the boundary field theory corresponds to the $\kappa/\ell \rightarrow 0$ limit in the bulk. Therefore, using (3.4) and (3.19) one can easily find

$$\Delta_L = \frac{a\kappa b^2}{16} , \quad \Delta_M = \frac{\kappa^2 b^2}{16} . \quad (3.20)$$

Let us consider the Cardy formula (3.3) and try to take its $\epsilon \rightarrow 0$ limit. For our current problem the relativistic central charges are $c_+ = c_- = 3\epsilon$. Using (3.20), we obtain

$$\begin{aligned} \lim_{\epsilon \rightarrow 0} S_{\text{CFT}} &= \lim_{\epsilon \rightarrow 0} 2\pi \left(\sqrt{\frac{c_+ \Delta_+}{6}} + \sqrt{\frac{c_- \Delta_-}{6}} \right) \\ &= \lim_{\epsilon \rightarrow 0} \pi \left[\sqrt{\epsilon \left(\frac{\Delta_M}{\epsilon} + \Delta_L \right)} + \sqrt{\epsilon \left(\frac{\Delta_M}{\epsilon} - \Delta_L \right)} \right] \\ &= 2\pi \sqrt{\Delta_M} = S_{\text{CCFT}} . \end{aligned} \quad (3.21)$$

This is the Cardy-like formula for the CCFT dual to the flat-space limit of NMG. Inserting (3.20) into (3.21), we finally recover the entropy (2.20),

$$S_{\text{CCFT}} = S_{\text{flat}} , \quad (3.22)$$

as we wanted to show. It is a quite nontrivial result since the theory has vanishing central charges.

4. CONCLUSIONS

In this paper, we have proposed a flat space generalization of the $\text{AdS}_3/\text{CFT}_2$ holographic correspondence.⁸ We have provided the first example of a holographic dual of an asymptoti-

⁷ The Hilbert space construction of a CCFT is analogous to that of the relativistic 2D CFT. Now, the states are labeled by the eigenvalues under L_0 and M_0 . We shall use the cylinder representation of the CCFT algebra [2]. The Hilbert space of the 2D CCFT are constructed by considering the states having definite scaling dimensions. We define primary states by demanding that the states in the theory be annihilated by all generators with $n > 0$. One can build up a tower of operators by acting on a primary state with the creation operators L_{-n} and M_{-n} ($n > 0$).

⁸ We note that in [28], the authors considered an asymptotically flat geometry which is a solution to three-dimensional Einstein gravity conformally coupled to a scalar field and discussed gravity/CFT correspondence for this background.

cally flat black hole solution. Due to the absence of black hole solutions in three-dimensional Einstein gravity with vanishing cosmological constant, we have considered higher-derivative gravity theories which admit asymptotically flat black hole solutions. The theory we have investigated is given by taking the flat-space limit ($\Lambda \rightarrow 0$) of NMG. We argued that the dual field theory of the black hole solution of this theory is a CCFT. For this purpose, we have constructed a stress tensor for the asymptotically flat black hole solution and computed the conserved charges. We then verified it using the first principle of black hole thermodynamics. Furthermore, we have used the Flat/CCFT correspondence to find the black hole entropy in terms of the asymptotic growth of the number of CCFT states.

It is interesting to note that the symmetry algebra of the corresponding CCFT had vanishing central charges though the asymptotic growth of states were nonzero. This remarkable point can be used for finding holographic duals of four-dimensional asymptotically flat spacetimes. According to the proposal of Flat/CCFT correspondence, the dual of four-dimensional asymptotically flat black holes are field theories with BMS_4 symmetry [25, 29]. In [30], the authors constructed the field-dependent central extension of BMS_4 algebra and found that for the Kerr black hole some of the charges involved divergent integrals on the 2-sphere if they used extended BMS algebra with both supertranslations and superrotations. Thus, at first sight, it seems that counting CCFT_3 states would be a problematic issue, but our current work shows that counting the asymptotic growth of CCFT states can be done whatever the central charges are.

Although our current study gives a holographic description of asymptotically flat black holes in three-dimensional higher-derivative gravity, we believe that the Flat/CCFT correspondence can be extended to find a holographic description of black holes in higher dimensions and, specifically, the four-dimensional Kerr black hole. We hope to explore other intriguing aspects of the relation between asymptotically flat spacetimes and CCFTs in our future works.

Acknowledgments

We would like to thank Ali Naseh for useful discussions. We are grateful to Álvaro Véliz-Orsorio and Arjun Bagchi for their useful comments on the revised version of the manuscript.

S.M.H. is supported in part by INFN.

- [1] A. Bagchi, “Correspondence between Asymptotically Flat Spacetimes and Nonrelativistic Conformal Field Theories,” *Phys. Rev. Lett.* **105**, 171601 (2010).
A. Bagchi, “The BMS/GCA correspondence,” arXiv:1006.3354 [hep-th].
- [2] A. Bagchi and R. Fareghbal, “BMS/GCA Redux: Towards Flatspace Holography from Non-Relativistic Symmetries,” *JHEP* **1210**, 092 (2012) [arXiv:1203.5795 [hep-th]].
- [3] C. Krishnan, A. Raju and S. Roy, “A Grassmann path from AdS_3 to flat space,” *JHEP* **1403**, 036 (2014) [arXiv:1312.2941 [hep-th]].
- [4] A. Bagchi, S. Detournay, R. Fareghbal and J. Simon, “Holography of 3d Flat Cosmological Horizons,” *Phys. Rev. Lett.* **110**, 141302 (2013) [arXiv:1208.4372 [hep-th]].
- [5] L. Cornalba and M. S. Costa, “A New cosmological scenario in string theory,” *Phys. Rev. D* **66**, 066001 (2002) [hep-th/0203031].
- [6] S. Detournay, “Inner Mechanics of 3d Black Holes,” *Phys. Rev. Lett.* **109**, 031101 (2012) [arXiv:1204.6088 [hep-th]].
A. Castro and M. J. Rodriguez, “Universal properties and the first law of black hole inner mechanics,” *Phys. Rev. D* **86**, 024008 (2012) [arXiv:1204.1284 [hep-th]].
- [7] R. Fareghbal and A. Naseh, “Aspects of Flat/CCFT Correspondence,” arXiv:1408.6932 [hep-th].
- [8] M. Riegler, “Flat space limit of higher-spin Cardy formula,” *Phys. Rev. D* **91**, no. 2, 024044 (2015) [arXiv:1408.6931 [hep-th]].
- [9] R. Fareghbal and A. Naseh, “Flat-Space Energy-Momentum Tensor from BMS/GCA Correspondence,” *JHEP* **1403**, 005 (2014) [arXiv:1312.2109 [hep-th]].
- [10] B. Czech, J. L. Karczmarek, F. Nogueira and M. Van Raamsdonk, “Rindler Quantum Gravity,” *Class. Quant. Grav.* **29**, 235025 (2012) [arXiv:1206.1323 [hep-th]].
- [11] M. Parikh and P. Samantray, “Rindler-AdS/CFT,” arXiv:1211.7370 [hep-th].
- [12] R. Fareghbal and A. Naseh, “Rindler/Contracted-CFT Correspondence,” *JHEP* **1406**, 134 (2014) [arXiv:1404.3937 [hep-th]].
- [13] A. Bagchi, R. Basu, D. Grumiller and M. Riegler, “Entanglement entropy in Galilean conformal field theories and flat holography,” *Phys. Rev. Lett.* **114**, no. 11, 111602 (2015)

- [arXiv:1410.4089 [hep-th]].
- [14] D. Ida, “No black hole theorem in three-dimensional gravity,” *Phys. Rev. Lett.* **85**, 3758 (2000) [gr-qc/0005129].
 - [15] R. M. Wald, “Black hole entropy is the Noether charge,” *Phys. Rev. D* **48**, 3427 (1993) [gr-qc/9307038].
 - [16] E. A. Bergshoeff, O. Hohm and P. K. Townsend, “Massive Gravity in Three Dimensions,” *Phys. Rev. Lett.* **102**, 201301 (2009) [arXiv:0901.1766 [hep-th]].
 - [17] S. Deser, “Ghost-free, finite, fourth order D=3 (alas) gravity,” *Phys. Rev. Lett.* **103**, 101302 (2009) [arXiv:0904.4473 [hep-th]].
 - [18] J. Oliva, D. Tempo and R. Troncoso, “Three-dimensional black holes, gravitational solitons, kinks and wormholes for BHT massive gravity,” *JHEP* **0907**, 011 (2009) [arXiv:0905.1545 [hep-th]].
 - [19] J. D. Brown and J. W. York, Jr., “Quasilocal energy and conserved charges derived from the gravitational action,” *Phys. Rev. D* **47**, 1407 (1993) [gr-qc/9209012].
 - [20] E. A. Bergshoeff, O. Hohm and P. K. Townsend, “More on Massive 3D Gravity,” *Phys. Rev. D* **79**, 124042 (2009) [arXiv:0905.1259 [hep-th]].
 - [21] G. Giribet, J. Oliva, D. Tempo and R. Troncoso, “Microscopic entropy of the three-dimensional rotating black hole of BHT massive gravity,” *Phys. Rev. D* **80**, 124046 (2009) [arXiv:0909.2564 [hep-th]].
 - [22] S. de Buyl, S. Detournay, G. Giribet and G. S. Ng, “Baby de Sitter black holes and dS_3/CFT_2 ,” *JHEP* **1402**, 020 (2014) [arXiv:1308.5569 [hep-th]].
 - [23] J. D. Brown and M. Henneaux, “Central Charges in the Canonical Realization of Asymptotic Symmetries: An Example from Three-Dimensional Gravity,” *Commun. Math. Phys.* **104**, 207 (1986).
 - [24] H. Bondi, M. G. van der Burg, and A. W. Metzner, “Gravitational waves in general relativity. 7. Waves from axisymmetric isolated systems,” *Proc. Roy. Soc. Lond. A* **269** (1962) 21.
R. K. Sachs, “Gravitational waves in general relativity. 8. Waves in asymptotically flat space-times,” *Proc. Roy. Soc. Lond. A* **270** (1962) 103.
R. K. Sachs, “Asymptotic symmetries in gravitational theory,” *Phys. Rev.* **128** (1962) 2851.
 - [25] G. Barnich and C. Troessaert, “Aspects of the BMS/CFT correspondence,” *JHEP* **1005**, 062 (2010) [arXiv:1001.1541 [hep-th]].

- [26] G. Barnich and G. Compere, “Classical central extension for asymptotic symmetries at null infinity in three spacetime dimensions,” *Class. Quant. Grav.* **24**, F15 (2007) [Erratum-ibid. **24**, 3139 (2007)] [arXiv:gr-qc/0610130].
- [27] Y. Kwon, S. Nam, J. D. Park and S. H. Yi, “Holographic Renormalization and Stress Tensors in New Massive Gravity,” *JHEP* **1111**, 029 (2011) [arXiv:1106.4609 [hep-th]].
- [28] M. Hasanpour, F. Loran and H. Razaghian, “Gravity/CFT correspondence for three dimensional Einstein gravity with a conformal scalar field,” *Nucl. Phys. B* **867**, 483 (2013) [arXiv:1104.5142 [hep-th]].
- [29] G. Barnich and C. Troessaert, “Symmetries of asymptotically flat 4 dimensional spacetimes at null infinity revisited,” *Phys. Rev. Lett.* **105**, 111103 (2010) [arXiv:0909.2617 [gr-qc]].
- [30] G. Barnich and C. Troessaert, “BMS charge algebra,” *JHEP* **1112**, 105 (2011) [arXiv:1106.0213 [hep-th]].